When gravity meets quantum mechanics: Nonunitary Newtonian Gravity at work in an optomechanical system

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In collaboration with
Filippo Maimone,
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“The extreme weakness of quantum gravitational effects now poses some philosophical problems: maybe nature is trying to tell us something new here, maybe we should not try to quantize gravity. It is possible perhaps that we should not insist on a uniformity of nature that would make everything quantized? Is it possible that gravity is not quantized and all the rest of the world is?”

“I would like to suggest that it is possible that quantum mechanics fails for large distances and large objects. Now, mind you, I do not say that quantum mechanics does fail at large distances, I only say that it is not inconsistent with what we do know. If this failure of quantum mechanics is connected with gravity, we might speculatively expect this to happen for masses such that $GM^2/\hbar c=1$, of $M$ near $10^{-5}$ grams, which corresponds to some $10^{18}$ particles.”

(R. P. Feynman, 1957)
Breakdown of predictability in gravitational collapse

S. W. Hawking

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(Received 25 August 1975)

The principle of equivalence, which says that gravity couples to the energy-momentum tensor of matter, and the quantum-mechanical requirement that energy should be positive imply that gravity is always attractive. This leads to singularities in any reasonable theory of gravitation. A singularity is a place where the classical concepts of space and time break down as do all the known laws of physics because they are all formulated on a classical space-time background. In this paper it is claimed that this breakdown is not merely a result of our ignorance of the correct theory but that it represents a fundamental limitation to our ability to predict the future, a limitation that is analogous but additional to the limitation imposed by the normal quantum-mechanical uncertainty principle. The new limitation arises because general relativity allows the causal structure of space-time to be very different from that of Minkowski space. The interaction region can be bounded not only by an initial surface on which data are given and a final surface on which measurements are made but also a “hidden surface” about which the observer has only limited information such as the mass, angular momentum, and charge. Concerning this hidden surface one has a “principle of ignorance”: The surface emits with equal probability all configurations of particles compatible with the observers limited knowledge. It is shown that the ignorance principle holds for the quantum-mechanical evaporation of black holes: The black hole creates particles in pairs, with one particle always falling into the hole and the other possibly escaping to infinity. Because part of the information about the state of the system is lost down the hole, the final situation is represented by a density matrix rather than a pure quantum state. This means there is no $S$ matrix for the process of black-hole formation and evaporation. Instead one has to introduce a new operator, called the superscattering operator, which maps density matrices describing the initial situation to density matrices describing the final situation.
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“...in order to gain a better understanding of the degrees of freedom responsible for black hole entropy, it will be necessary to achieve a deeper understanding of the notion of entropy itself. Even in flat space-time, there is far from universal agreement as to the meaning of entropy – particularly in quantum theory – and as to the nature of the second law of thermodynamics.”

(R. M. Wald, 2001)
Introduction: the Quantum Measurement Problem

Role of gravity in the wave function collapse: theory

A gedanken experiment by R. Penrose

A theoretical proposal: Nonunitary Newtonian Gravity (NNG)

Application of NNG model to Penrose experiment

Prospects for future experimental tests

Conclusions and perspectives
Outline

- Introduction: the Quantum Measurement Problem
- Role of gravity in the wave function collapse: theory
- A gedanken experiment by R. Penrose
- A theoretical proposal: Nonunitary Newtonian Gravity (NNG)
- Application of NNG model to Penrose experiment
- Prospects for future experimental tests
- Conclusions and perspectives
W. H. Zurek: Decoherence and the transition from Quantum to Classical
Introduction: the Quantum Measurement Problem
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Experiments tell us that, if the selected states of the measuring apparatus are in one-to-one correspondence with the basis $|\psi_n\rangle$, after the measurement the quantum system is found to be in one of the eigenstates, say $|\psi_n\rangle$. Repeated measurements on identical copies of the quantum system show that the system is found to be in one or the other eigenstates $|\psi_n\rangle$, with the probability to be found in state $|\psi_n\rangle$ being given by $|a_n|^2$ (BORN PROBABILITY RULE).
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The transition $|\psi\rangle \rightarrow |\psi_n\rangle$ CANNOT be described by Schroedinger equation, which is linear and preserve superpositions.
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Quantum mechanics and Schroedinger equation must be modified

QUANTUM MEASUREMENT PROBLEM: what is the correct physical description of this measurement process and of the observed result?

AND: how classicality can emerge from the quantum world?
An overview on possible modifications:
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1. **Models that do not involve gravity**: the wave function collapse could be explained as a dynamical reduction process using stochastic differential equations. So the Schroedinger equation is augmented by a stochastic term which could induce collapse.

2. **Models that involve gravity**: the gravitational field produced by a classical object obeys the laws of general relativity, or in the limiting case, those of Newtonian gravity. Since the position of the classical object is subject to tiny quantum uncertainties, the gravitational field and the curvature tensor produced by it are also subject to quantum fluctuations. More explicitly, the principle of general covariance and the principle of linear superposition in quantum mechanics are in conflict with each other.
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\[ |\Xi\rangle = w|\psi_1\rangle + z|\psi_2\rangle \]

\[ i\hbar \frac{\partial}{\partial t} |\psi_i\rangle = E|\psi_i\rangle, \quad i = 1,2 \]

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If extended to the gravitational fields produced by the body’s own energy-momentum distribution, Superposition Principle no longer holds:
What is the meaning of superposing two space-times?

In a stationary space-time, i.e. Killing vector $\frac{\partial}{\partial t}$ is time-like.

Within GR, gauge invariance implies a complete loss of identity of points. On the other hand in QM one could not be able to construct all the many wave functions for a particle (i.e. superpositions of different position states of the particle) if points are ultimately the same point.
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Roger Penrose proposal:
“...when the geometries become significantly different from each other, we have no absolute means of identifying a point in one geometry with any particular point in the other..., so the very idea that one could form a superposition of the matter states within these two separate spaces becomes profoundly obscure” (R. Penrose).

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Roger Penrose proposal:

$\varphi_1 + \sqrt{2} \Psi \varphi_2$

Penrose considers a balanced superposition of two separate wave packets representing two different positions of a massive object. If the mass $M$ is large enough, the two wave packets represent two very different mass distributions (lumps). By assuming that in each space-time we can use the notions of stationarity and energy, while the difference between the identified time translation operators gives a measure of the ill-definiteness or uncertainty of the final superposition’s energy, the decay time for the balanced superposition of two lumps is:
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How to compare two space-times in a given region?

By comparing the behavior of the geodesics in that region.
Minimalist Proposal (no dynamical model)!!!
If $a$ and $a'$ are the 3-accelerations of the lump in the two positions and $\Phi$ and $\Phi'$ are the corresponding Newtonian potentials, the self-gravitational energy of the difference between the mass distributions of each of the two lump locations is:

$$\Delta E_{grav} = G^{-1} \int d^3 x (a - a')^2 = G^{-1} \int d^3 x (\nabla \Phi - \nabla \Phi')^2$$

$$= -G^{-1} \int d^3 x (\Phi - \Phi') (\nabla^2 \Phi - \nabla^2 \Phi')$$

$$= -G \iint d^3 x d^3 y \frac{(\rho(x) - \rho'(x))(\rho(y) - \rho'(y))}{|x - y|}$$

**Minimalist Proposal (no dynamical model)!!!**

**PROBLEM:** since for atomic systems $\Delta E_{grav}$ is very small, the decay time is very long, i.e. atomic superpositions will never decay. On the other hand, massive superpositions would never be formed because they would decay immediately.
Non linear equations
SOLUTION: the basic stationary states into which a superposition of such states is to decay are considered as stationary solutions of a Schroedinger equation augmented by an additional term provided by a certain gravitational potential. This potential is the one which arises from the mass density given by the expectation value of the mass distribution in the state determined by the wave function. In general it would be sufficient to consider Newtonian gravity. The result is:

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-\frac{\hbar^2}{2m} \nabla^2 \psi + U\psi = E\psi \\
\nabla^2 U = 4\pi Gm^2 |\psi|^2
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SOLUTION: the basic stationary states into which a superposition of such states is to decay are considered as stationary solutions of a Schrödinger equation augmented by an additional term provided by a certain gravitational potential. This potential is the one which arises from the mass density given by the expectation value of the mass distribution in the state determined by the wave function. In general it would be sufficient to consider Newtonian gravity. The result is:

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R. Penrose, the FELIX (Free-orbit Experiment with Laser Interferometry X-rays)
A gedanken experiment by R. Penrose

- M (= 5 \cdot 10^{-12}\text{kg}) in a Schrödinger’s cat state by a X photon in a superposition of two beams maintained in coherence;
- M rigid and at initial position in 1/10 sec;
- Experiment in the space to maintain the coherence for this time and avoid interaction with matter;
- Photon and mirror are entangled: If $O.R. \Rightarrow D$ is activated 50% of the times.

R. Penrose, the FELIX (Free-orbit Experiment with Laser Interferometry X-rays)

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Gedanken experiment in detail

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L. Hardy, Oxford University preprint (1998)
A beam splitter is placed in the path of an incident photon emanating from a source S. A horizontally movable mass M is attached to the wall opposite to S by means of a restoring device with a spring constant k. There are two reflecting mirrors, one of them affixed on the mass and the other one at the end of the vertical arm of the interferometer, both being at an exactly equal distance from the beam splitter. The earth provides a frame of reference, and the final destination of interest for the photon is the detector D.
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The transformations of the photon states due to the beam splitter are:

\[
|a \pm \rangle \leftrightarrow \frac{1}{\sqrt{2}} \left( |b \pm \rangle + |c \pm \rangle \right)
\]

\[
|d \pm \rangle \leftrightarrow \frac{1}{\sqrt{2}} \left( |c \pm \rangle - |b \pm \rangle \right)
\]

Initial state:  \[
|a + \rangle \otimes |M0\rangle
\]

Incident photon  Mass
The photon can follow two paths, the vertical one or the horizontal one. The net result of these two alternatives in the presence of the beam splitter, quantum mechanically is encoded in the state:

Each of the two options would lead the photon back towards the beam splitter, so the above composite state will evolve into:
As the photon passes through the beam splitter, this initial state evolves into:

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\frac{1}{\sqrt{2}} \left( (b + \rangle + |c + \rangle \right) \otimes |M0\rangle
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The photon can follow two paths, the vertical one or the horizontal one. The net result of these two alternatives in the presence of the beam splitter, quantum mechanically is encoded in the state:

\[
\frac{1}{\sqrt{2}} \left( |b-\rangle \otimes |M+\rangle + |c-\rangle \otimes |M0\rangle \right)
\]

Each of the two options would lead the photon back towards the beam splitter, so the above composite state will evolve into:

\[
\frac{1}{2} \left( \{a-\} - |d-\rangle \right) \otimes |M+\rangle + \{a-\} + |d-\rangle \otimes |M0\rangle
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\[
\frac{1}{2} \left( |b+\rangle + |c+\rangle \right) \otimes \left( |M0\rangle - |M-\rangle \right)
\]
But let us suppose that the macroscopic superposition of the mass \((1/\sqrt{2})[|M_0>-|M_+>]\) has undergone a Penrose-type state reduction. In this case the state of the mass just before the second photon is sent in would be a proper mixture of \(|M_0>\) and \(|M->\).
After a recoil of the photon from the two mirrors, the state will become:

\[
\frac{1}{2} \left[ \left( b - \right) \otimes \left| M^+ \right> + \left( c - \right) \otimes \left| M^0 \right> - \left( b - \right) \otimes \left| M^0 \right> - \left( c - \right) \otimes \left| M^- \right> \right]
\]

Finally, after the evolution of the photon back through the beam-splitter, we get:

\[
\frac{1}{2\sqrt{2}} \left( a - \right) \otimes \left| M^+ \right> + \frac{1}{\sqrt{2}} \left( d - \right) \otimes \left| M^0 \right> - \frac{1}{2\sqrt{2}} \left( a - \right) \otimes \left| M^- \right>
\]

Following QM, the probability of detecting a photon in the detector D is 75%.

But let us suppose that the macroscopic superposition of the mass \((1/\sqrt{2})[|M^0\rangle - |M^+\rangle]\) has undergone a Penrose-type state reduction. In this case the state of the mass just before the second photon is sent in would be a proper mixture of \(|M^0\rangle\) and \(|M^-\rangle\).
As the photon is reflected off the two mirrors and passed again through the beam-splitter, these two alternatives will evolve independently into the final disjoint state:

Thus, the overall disjoint state after the photon has passed through the beam splitter is:

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\[ \frac{1}{\sqrt{2}} \left( (b+) + |c+\rangle \right) \otimes |M0\rangle \quad \text{or} \quad \frac{1}{\sqrt{2}} \left( (b+) + |c+\rangle \right) \otimes |M-\rangle \]

As the photon is reflected off the two mirrors and passed again through the beam-splitter, these two alternatives will evolve independently into the final disjoint state:

\[ \frac{1}{2} \left[ \left( (a-) - |d-\rangle \right) \otimes |M+\rangle + \left( (a-) + |d-\rangle \right) \otimes |M0\rangle \right] \quad \text{or} \quad \frac{1}{2} \left[ \left( (a-) - |d-\rangle \right) \otimes |M-\rangle + \left( (a-) + |d-\rangle \right) \otimes |M0\rangle \right] \]
Thus, the overall disjoint state after the photon has passed through the beam splitter is:

\[
\frac{1}{\sqrt{2}} \left( (b+) + (c+) \right) \otimes |M0\rangle
\]

or

\[
\frac{1}{\sqrt{2}} \left( (b+) + (c+) \right) \otimes |M-\rangle
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\]

or

\[
\frac{1}{2} \left[ \left( (a-) - (d-) \right) \otimes |M0\rangle + \left( (a-) + (d-) \right) \otimes |M-\rangle \right]
\]
Thus, the overall disjoint state after the photon has passed through the beam splitter is:

\[
\frac{1}{\sqrt{2}} \left( |b+\rangle + |c+\rangle \right) \otimes |M0\rangle \quad \text{or} \quad \frac{1}{\sqrt{2}} \left( |b+\rangle + |c+\rangle \right) \otimes |M-\rangle
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\frac{1}{2} \left[ \left( |a-\rangle - |d-\rangle \right) \otimes |M+\rangle + \left( |a-\rangle + |d-\rangle \right) \otimes |M0\rangle \right] \quad \text{or} \quad \frac{1}{2} \left[ \left( |a-\rangle - |d-\rangle \right) \otimes |M0\rangle + \left( |a-\rangle + |d-\rangle \right) \otimes |M-\rangle \right]
\]

If Penrose’s proposal is correct, then, after the photon passes through the beam splitter second time around, it would go to the detector only 50% of the time and not 75% of the time as QM predict.

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A proposal of experimental realization

This proposal considers the relatively small CM-displacement of a lump of $10^{14}$ proton masses in an interferometric device in which two high-finesse optical cavities are inserted into its arms. The cavity in arm A has a very small end mirror mounted on a micro-mechanical oscillator (cantilever), which suffers the radiation pressure of the photon confined inside it and as a consequence can be excited into a distinguishable quantum state. A single photon incident on a 50-50 beam splitter will realize a superposition of being in either of the two arms; then, the coupling between the photon and the cantilever will lead to an **entangled state** putting the cantilever into a superposition of distinct positions. After a full mechanical period of the cantilever, it recovers the original position; if the photon leaks out of the cavity at this stage, a revival of the interference (visibility) is observed, provided that the quantum superposition state of the system survives at the intermediate times. Conversely, if the state of the system collapses due to some decoherence mechanism, visibility will not revive.
This proposal considers the relatively small CM-displacement of a lump of $10^{14}$ proton masses in an interferometric device in which two high-finesse optical cavities are inserted into its arms. The cavity in arm A has a very small end mirror mounted on a micro-mechanical oscillator (cantilever), which suffers the radiation pressure of the photon confined inside it and as a consequence can be excited into a distinguishable quantum state. A single photon incident on a 50-50 beam splitter will realize a superposition of being in either of the two arms; then, the coupling between the photon and the cantilever will lead to an entangled state putting the cantilever into a superposition of distinct positions. After a full mechanical period of the cantilever, it recovers the original position; if the photon leaks out of the cavity at this stage, a revival of the interference (visibility) is observed, provided that the quantum superposition state of the system survives at the intermediate times. Conversely, if the state of the system collapses due to some decoherence mechanism, visibility will not revive.
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A measurement of the magnitude of the revival of visibility gives a measurement of decoherence occurred in the time interval under consideration.
Optomechanical coupling constant

Unitary evolution operator
\[ H = \hbar \omega_a \left[ a^+ a + b^+ b \right] + \hbar \omega_c \left[ c^+ c - \kappa a^+ a (c + c^+) \right] \]

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- **Effective Hamiltonian**
- **Frequency of the optical field**
- **Optomechanical coupling constant**
- **Unitary evolution operator**
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**Effective Hamiltonian**

**Frequency of the optical field**

**Operators for photons in the arms A and B of the interferometer**

**Optomechanical coupling constant**

**Unitary evolution operator**


\[ H = \hbar \omega_a \left[ a^+ a + b^+ b \right] + \hbar \omega_c \left[ c^+ c - \kappa a^+ a (c + c^+) \right] \]

- **Effective Hamiltonian**
- **Frequency of the optical field**
- **Operators for photons in the arms A and B of the interferometer**
- **Mechanical frequency of the cantilever**
- **Optomechanical coupling constant**
- **Unitary evolution operator**

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\[ H = \hbar \omega_a a^+ a + b^+ b + \hbar \omega_c c^+ c - \kappa a^+ a (c + c^+) \]

- **Effective Hamiltonian**
- **Frequency of the optical field**
- **Operators for photons in the arms A and B of the interferometer**
- **Mechanical frequency of the cantilever**
- **Phonon operators for the vibrational mode of the cantilever**
- **Optomechanical coupling constant**
- **Unitary evolution operator**
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**Effective Hamiltonian**

**Frequency of the optical field**

**Operators for photons in the arms A and B of the interferometer**

**Mechanical frequency of the cantilever**

**Phonon operators for the vibrational mode of the cantilever**

**Optomechanical coupling constant**

\[ \kappa = \frac{\omega_a}{L \omega_c} \sqrt{\frac{\hbar}{2 m \omega_c}} = \frac{\sqrt{2} N x_0}{\lambda} \]
The effective Hamiltonian is given by:

\[ H = \hbar \omega_a \left( a^+ a + b^+ b \right) + \hbar \omega_c \left( c^+ c - \kappa a^+ a (c + c^+) \right) \]

The frequency of the optical field is denoted by \( \omega_a \) and the mechanical frequency of the cantilever by \( \omega_c \).

The optomechanical coupling constant \( \kappa \) is given by:

\[ \kappa = \frac{\omega_a}{L \omega_c} \sqrt{\frac{\hbar}{2m \omega_c}} = \frac{\sqrt{2Nx_0}}{\lambda} \]

Operators for photons in the arms A and B of the interferometer are denoted by \( a \) and \( b \).

Phonon operators for the vibrational mode of the cantilever are denoted by \( c \).

Unitary evolution operator \( U(t) \) is given by:

\[
U(t) = \exp \left[ -i \omega_a t \left( a^+ a + b^+ b \right) - i \left( \kappa a^+ a \right)^2 \left( \omega_c t - \sin \omega_c t \right) \right] \exp \left[ \kappa a^+ a \left( 1 - e^{-i \omega_c t} \right) c^+ - \left( 1 - e^{i \omega_c t} \right) c \right] \exp \left[ -i \omega_c c^+ c t \right]
\]
Initial state:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \left( |0,1\rangle_{n_a,n_b} + |1,0\rangle_{n_a,n_b} \right) \otimes |\beta_c\rangle$$
Initial state:

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Unitary evolution

\[ |\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_a t} \left( |0,1\rangle \otimes |\beta e^{-i\omega_c t}\rangle + e^{i\kappa^2 (\omega_c t - \sin(\omega_c t)) + i\kappa \im \beta \left( 1 - e^{-i\omega_c t} \right)} |1,0\rangle \otimes |\kappa \left( 1 - e^{-i\omega_c t} \right) + |\beta e^{-i\omega_c t}\rangle \right) \]
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Unitary evolution

\[ |\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_a t} \left( |0,1\rangle \otimes |\beta e^{-i\omega_c t}\rangle \right) + \]

\[ e^{i\kappa^2 (\omega_c t - \sin(\omega_c t)) + i\kappa \Im \left[ \beta \left( 1 - e^{-i\omega_c t} \right) \right]} |1,0\rangle \otimes |\kappa \left( 1 - e^{-i\omega_c t} \right) + \beta e^{-i\omega_c t}\rangle \]
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\[ |\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_a t} \left( |0,1\rangle \otimes |\beta e^{-i\omega_c t}\rangle + e^{i\kappa^2 (\omega_c t - \sin(\omega_c t)) + i\kappa \text{ Im} \left[ \beta \left( 1 - e^{-i\omega_c t} \right) \right]} |1,0\rangle \otimes |\kappa (1 - e^{-i\omega_c t}) + \beta e^{-i\omega_c t}\rangle \right) \]

\[ \nu(t) = e^{-\kappa^2 (1 - \cos(\omega_c t))} \]
Initial state:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \left( |0,1\rangle_{n_a,n_b} + |1,0\rangle_{n_a,n_b} \right) \otimes |\beta_c\rangle$$

Unitary evolution

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_a t} \left( |0,1\rangle \otimes |\beta e^{-i\omega_c t}\rangle + e^{i\kappa^2 (\omega_c t - \sin(\omega_c t)) + i\kappa \Im[\beta (-e^{-i\omega_c t})]} |1,0\rangle \otimes |\kappa (1 - e^{-i\omega_c t}) + \beta e^{-i\omega_c t}\rangle \right)$$

Interferometric visibility as seen by the two single photon detectors:

$$\nu(t) = e^{-\kappa^2 (1 - \cos(\omega_c t))}$$

Mirror Coherent state
The result is a periodic behavior characterized by a suppression of the interference visibility after half a mechanical period and a revival of perfect visibility after a full period, provided there is no decoherence in the state of the cantilever.
A theoretical proposal: Nonunitary Newtonian Gravity (S. De Filippo)

PROBLEM: How to construct a nonunitary theory incorporating gravity? We look for a minimal modification of quantum dynamics, so we refer to the low-energy, low temperature physics, where a non-relativistic setting is appropriate.
A theoretical proposal: Nonunitary Newtonian Gravity (S. De Filippo)

PROBLEM: How to construct a nonunitary theory incorporating gravity? We look for a minimal modification of quantum dynamics, so we refer to the low-energy, low-temperature physics, where a non-relativistic setting is appropriate.

BASIC REQUIREMENTS:
• Consistency with the basic formal relations of QM (canonical commutation relations) must be preserved by the equation of motion.
• Violations of energy-momentum conservation laws of causality must eventually be as small as to be compatible with ordinary laboratory physics.
• The nonunitary terms in the equation of motion for the density operator $\rho$ must have a gravitational origin.
• In the classical limit the Newtonian gravitation law has to be recovered while macroscopic gravitational fields must enter the Schroedinger equation like a Coulomb field.
• Consistency with Statistical Mechanics is required.
• The dynamics must reduce superpostions of different space-times mass distributions and must be compatible with the microscopic wave like properties of particles as described by the Schroedinger equation.

While reproducing at a macroscopic level the ordinary Newtonian interaction, the NNG model shows a mass threshold for gravitational localization which, for ordinary matter density, is about $10^{11}$ proton masses.

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The most general meta-Hamiltonian compatible with the ordinary classical Newtonian gravity is:

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The most general meta-Hamiltonian compatible with the ordinary classical Newtonian gravity is:

$$H_{tot} = \sum_{j=1}^{N} H\left[\psi_{j}^{+},\psi_{j}\right] - \frac{m^{2}G}{2N} \sum_{j=1}^{N} \int dx dy \frac{\psi_{j}^{+}(x)\psi_{j}(x)\psi_{j}^{+}(y)\psi_{j}(y)}{|x-y|} (1 - \varepsilon)$$

$$- \frac{m^{2}G}{N} \sum_{j \neq k}^{N} \int dx dy \frac{\psi_{j}^{+}(x)\psi_{j}(x)\psi_{k}^{+}(y)\psi_{k}(y)}{|x-y|} \left(1 + \frac{\varepsilon}{N-1}\right)$$
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• G and m are the gravitational constant and the mass, while \( y_1, y_2, \ldots, y_N \) are \( N \) equivalent commuting copies of the one particle annihilation operator.
• One of the \( N \) operator algebras, say \( j=1 \), is identified with the observable algebra while the others are the hidden degrees of freedom.
• Physical states are obtained from meta-states by tracing out hidden degrees of freedom.
• The model is nonlinear and strongly non-Markovian.
• The model depends on two parameters, \( N \) and \( e \). \( N \) is the number of replicas in interaction while \( e \) modulates the degree of nonunitarity encoded in the gravitational interaction.
• The limit \( N \to \infty, e=1 \) reproduces the Newton-Schrödinger equation.

On the basis of thermodynamic requirements we choose \( N = 2 \) and maximize the effect of nonunitarity by fixing \( e = 1 \). So we get the following meta-Hamiltonian:
\[ H_{tot} = H \left[ \Psi^+, \Psi \right] + H \left[ \bar{\Psi}^+, \bar{\Psi} \right] \]

\[ - G \sum_{j,k}^{N} m_j m_k \int dx dy \frac{\psi_j^+ (x) \psi_j (x) \bar{\psi}_k^+ (y) \bar{\psi}_k (y)}{|x-y|} \]
$$H_{tot} = H [\Psi^+, \Psi] + H [\bar{\Psi}^+, \bar{\Psi}]$$
$$- G \sum_{j,k}^{N} m_j m_k \int dx dy \frac{\psi_j^+ (x) \psi_j (x) \bar{\Psi}^+_k (y) \bar{\Psi}_k (y)}{|x - y|}$$

$$F_{\Psi} \otimes F_{\bar{\Psi}}$$
$$\left| 0 \right> = \left| 0 \right>_{\Psi} \otimes \left| 0 \right>_{\bar{\Psi}}$$
\[ H_{\text{tot}} = H \left[ \psi^+, \psi \right] + H \left[ \tilde{\psi}^+, \tilde{\psi} \right] - G \sum_{j,k}^N m_j m_k \int dx dy \frac{\psi_j^+(x) \psi_j(x) \tilde{\psi}_k^+(y) \tilde{\psi}_k(y)}{|x - y|} \]

\[ F_{\psi} \otimes F_{\tilde{\psi}} \]

\[ \left| 0 \right\rangle = \left| 0 \right\rangle_\psi \otimes \left| 0 \right\rangle_{\tilde{\psi}} \]

Fock space
\[ H_{tot} = H\left[ \psi^+, \psi \right] + H\left[ \bar{\psi}^+, \bar{\psi} \right] \]

\[- G \sum_{j,k}^{N} m_j m_k \int dx dy \frac{\psi^+_j(x) \psi_j(x) \bar{\psi}^+_k(y) \bar{\psi}_k(y)}{|x - y|} \]

\[ F_\psi \otimes F_{\bar{\psi}} \]

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Fock space

Vacuum

General meta-state corresponding to one particle states
\[ H_{tot} = H \left[ \psi^+, \psi \right] + H \left[ \bar{\psi}^+, \bar{\psi} \right] \]
\[ - G \sum_{j,k} m_j m_k \int dx dy \frac{\psi_j^+(x) \psi_j(x) \bar{\psi}_k^+(y) \bar{\psi}_k(y)}{|x - y|} \]

\[ F_\psi \otimes F_{\bar{\psi}} \]
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Fock space
Vacuum

General meta-state corresponding to one particle states

\[ \langle f | = \int dx \int dy f(x, y) \psi_j^+(x) \bar{\psi}_j^+(y) |0\rangle \]
\[ f(x, y) = f(y, x) \]
\[ H_{tot} = H\left[\psi^+,\psi\right] + H\left[\bar{\psi}^+,\bar{\psi}\right] \]

\[- G \sum_{j,k}^{N} m_j m_k \int dx dy \frac{\psi_j^+(x)\psi_j(x)\bar{\psi}_k^+(y)\bar{\psi}_k(y)}{|x-y|} \]

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\[ \left\| 0 \right\rangle = \left\| 0 \right\rangle_\psi \otimes \left\| 0 \right\rangle_{\bar{\psi}} \]

\[ \left\| f \right\rangle \rangle = \int dx \int dy f(x, y)\psi_j^+(x)\bar{\psi}_j^+(y)\left\| 0 \right\rangle \]

\[ f(x, y) = f(y, x) \]

Fock space
Vacuum

General meta-state corresponding to one particle states

General meta-state corresponding to n-particle states
\[ H_{tot} = H \left[ \psi^+ , \psi \right] + H \left[ \bar{\psi}^+ , \bar{\psi} \right] \]
\[- G \sum^{N}_{j,k} m_j m_k \int dx dy \frac{\psi^+_j(x) \psi^+_j(y) \bar{\psi}^+_{k}(x) \bar{\psi}^+_{k}(y)}{|x - y|} \]

\[ F_\psi \otimes F_{\bar{\psi}} \]

\[ \left| 0 \right\rangle = \left| 0 \right\rangle_\psi \otimes \left| 0 \right\rangle_{\bar{\psi}} \]

**Fock space**

**Vacuum**

**General meta-state corresponding to one particle states**

**General meta-state corresponding to n-particle states**

\[ \left| f \right\rangle = \int dx \int dy f(x, y) \psi^+_j(x) \bar{\psi}^+_j(y) \left| 0 \right\rangle \]
\[ f(x, y) = f(y, x) \]

\[ \int d^n x \int d^n y \left( g(x_1, \ldots, x_n) g(y_1, \ldots, y_n) \times \psi^+_{j_1}(x_1) \cdots \psi^+_{j_n}(x_n) \bar{\psi}^+_{j_1}(y_1) \cdots \bar{\psi}^+_{j_n}(y_n) \left| 0 \right\rangle \right) \]
Orders of magnitude for identification of NNG effects in the experiment

SOLUTION: If $M<M_{tr}$ the metasystem behaves like an hydrogen-like system, while in the case $M>M_{tr}$ the hidden mass-copy is quite well superposed to the physical one.
Orders of magnitude for identification of NNG effects in the experiment

PROBLEM: How to identify the threshold mass of localization $M_{tr} \sim 10^{11}$ proton masses when considering self-interaction gravitational energy?

SOLUTION: If $M < M_{tr}$ the metasystem behaves like an hydrogen-like system, while in the case $M > M_{tr}$ the hidden mass-copy is quite well superposed to the physical one.

The interaction potential can be approximated, within the lowest energy state of the meta-system, by the harmonic oscillator ground state with gaussian wave function width:

$$\Lambda_G = \left( \frac{\hbar}{\sqrt{(4/3)\pi G \rho_{sil} M^2}} \right)^{1/2}, \quad \rho_{sil} = 5 \times 10^3 \text{ Kg} / \text{m}^3$$
Wave packets separation
Wave packets separation

$$\Delta x = \kappa \sqrt{\frac{\hbar}{2M\omega_m}}$$

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Wave packets separation

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Optomechanical coupling constant
Wave packets separation

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Optomechanical coupling constant

Mirror frequency
For NNG to be effective in localizing the mirror the condition $\Delta x > \sim \Lambda_G$ must be fulfilled, that is:

$$\Delta x = \kappa \sqrt{\frac{\hbar}{2M\omega_m}}$$
For NNG to be effective in localizing the mirror the condition $\Delta x > \Lambda_G$ must be fulfilled, that is:

$$\chi \equiv \frac{1}{\rho_{sil} G \left( \frac{\omega_m}{\kappa} \right)^2} \leq 1$$
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$$\omega_m \approx 2\pi \times 500 \text{Hz}$$

$$\rho_{sil} \approx 5 \times 10^3 \text{Kg} / m^3$$

$$\kappa \approx 1$$
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$$\kappa \approx 1$$

$$\Delta x \approx 5.79 \times 10^{-14} \text{m}$$
$$\chi \approx 10^{13}$$
An enhancement in the possibility of observing gravitational decoherence effects is provided by taking into account the real distribution of mass inside a crystal, which is very concentrated within nuclei (L. Diosi, J. Phys. A: Math. Theor. 40 (2007) 2989).

Granular matter distribution

\[ \rho_{nucl} \approx 10^4 \rho_{sil} \]
\[ \chi \approx 10^9 \]

COMMENT: when the relative displacement of meta-masses \( \Delta x \) is of the order of the nucleous effective radius, granularity has to be considered; when \( \Delta x \) is made much greater than interatomic separation, homogeneity assumption is suitable; finally, for \( \Delta x \) of the order of interatomic separation, if imperfections are present in the sample, then meta-masses are likely to feel an effective homogeneous masses potential; otherwise, for a really perfect crystal granularity should come again into play.
Decoherence rate must be at least comparable with (or lower than) a period of natural oscillation of the mirror, which, in turn, must be comparable with (or lower than) environmental decoherence rate for the experiment to be feasible.
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\[ \frac{E_G}{\hbar} \approx \frac{\pi k^2 \hbar G \rho_{sil} \rho_{nucl}}{3\omega_m} \approx \omega_m \geq \gamma_D \]
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Gravitational interaction energy of meta-masses

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Gravitational interaction energy of meta-masses

Enviromental decoherence rate of the mirror (exp. par.)
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- Gravitational interaction energy of meta-masses
- Environmetal decoherence rate of the mirror (exp. par.)

**Putting experimental parameters**
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Gravitational interaction energy of meta-masses

Environmetal decoherence rate of the mirror (exp. par.)

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\[
\frac{E_G}{\hbar} \approx \frac{\pi \kappa^2 \hbar G \rho_{\text{sil}} [\rho_{\text{nucl}}]}{3 \omega_m} \approx \omega_m \geq \gamma_D
\]

Gravitational interaction energy of meta-masses

Enviromental decoherence rate of the mirror (exp. par.)

Putting experimental parameters

\[\omega_m = \omega_m^{\text{exp}} \approx 2\pi \times 500 \text{ Hz}\]
Calculation of interference visibility

Gravity-free Hamiltonian:

Total meta-Hamiltonian:
Calculation of interference visibility

\textbf{Gravity-free Hamiltonian:}

\[ H_{\text{free}} \left[ b, b^+, N_A, N_B; \omega_m \right] = \hbar \omega_{Ph} (N_A + N_B) + \hbar \omega_m b^+ b - \hbar g N_A (b + b^+) \]

\[ g = \kappa \omega_m \]

\textbf{Total meta-Hamiltonian:}
Calculation of interference visibility

**Gravity-free Hamiltonian:**

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g = \kappa \omega_m
\]

**Total meta-Hamiltonian:**

\[
H_{\text{tot}} = H_{\text{free}} \left[ b, b^+, N_A; \omega_m^* \right] + H_{\text{free}} \left[ \tilde{b}, \tilde{b}^+, \tilde{N}_A; \omega_m^* \right] - K_G \left( b + b^+ \right) \left( \tilde{b} + \tilde{b}^+ \right) \\
\omega_m^* = \omega_m \sqrt{1 + 2 \frac{K_G}{\hbar \omega_m}}
\]
Calculation of interference visibility

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\[ H_{\text{free}} \left[ b, b^+, N_A, N_B; \omega_m \right] = \hbar \omega_{\text{Ph}} (N_A + N_B) + \hbar \omega_m b^+ b - \hbar g N_A (b + b^+) \]

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\[ \omega_m^* = \omega_m \sqrt{1 + 2 \frac{K_G}{\hbar \omega_m}} \]

Gravitational coupling strength in the homogeneous case
Calculation of interference visibility

Gravity-free Hamiltonian:
\[
H_{\text{free}} b, b^+, N_A, N_B; \omega_m \rightleftharpoons \hbar \omega_{ph} (N_A + N_B) + \hbar \omega_m b^+ b - \hbar g N_A (b + b^+)
\]
\[g = K \omega_m\]

Total meta-Hamiltonian:
\[
H_{\text{tot}} = H_{\text{free}} b, b^+, N_A; \omega_m^* + H_{\text{free}} \tilde{b}, \tilde{b}^+, \tilde{N}_A; \omega_m^* \rightleftharpoons K_G (b + b^+) (\tilde{b} + \tilde{b}^+)
\]
\[\omega_m^* = \omega_m \sqrt{1 + 2 \frac{K_G}{\hbar \omega_m}}\]

Gravitational coupling strength in the homogeneous case
\[K_G^{\text{hom}} = \frac{\pi \hbar G \rho_{\text{sil}}}{3 \omega_m}\]
Let us study the dynamical evolution of the meta-system:
Gravitational coupling strength in the granular case

Let us study the dynamical evolution of the meta-system:
Gravitational coupling strength in the granular case

\[ K^{\text{gran}}_G = \frac{2\hbar G \rho_{\text{nucl}}}{3 \omega_m} \approx 10^4 \times K^\text{hom}_G \]

Let us study the dynamical evolution of the meta-system:
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Let us study the dynamical evolution of the meta-system:
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\[
|\Psi(0)\rangle = \frac{1}{2} \left[ \left( |0_A 1_B\rangle + |1_A 0_B\rangle \right) \alpha \langle \right] \bigotimes \left[ \left( |0_A 1_B\rangle + |1_A 0_B\rangle \right) \tilde{\alpha} \langle \right] = |\psi(0)\rangle \bigotimes |\psi(0)\rangle
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Let us study the dynamical evolution of the meta-system:

Initial meta-state

\[ ||\Psi(0)|| = \frac{1}{2} \left[ (|0_{A}1_{B}\rangle + |1_{A}0_{B}\rangle)\alpha\right] \otimes \left[ (|0_{\tilde{A}}1_{\tilde{B}}\rangle + |1_{\tilde{A}}0_{\tilde{B}}\rangle)\tilde{\alpha}\right] = |\Psi(0)\rangle \otimes |\Psi(0)\rangle \]

Schroedinger state at time t
Gravitational coupling strength in the granular case

\[ K_G^{\text{gran}} = \frac{2\hbar G \rho_{\text{nucl}}}{3\omega_m} \approx 10^4 \times K_G^{\text{hom}} \]

Let us study the dynamical evolution of the meta-system:

**Initial meta-state**

\[ \left\langle \Psi(0) \right\rangle = \frac{1}{2} \left[ \left( 0 \_A 1 \_B \right) + 1 \_A 0 \_B \right) \alpha \right\rangle \times \left[ \left( 0 \_\tilde{A} 1 \_\tilde{B} \right) + 1 \_\tilde{A} 0 \_\tilde{B} \right) \tilde{\alpha} \right\rangle = \left\langle \Psi(0) \right\rangle \otimes \left\langle \Psi(0) \right\rangle \]

**Schroedinger state at time t**

\[ \left\langle \Psi(t) \right\rangle = \frac{1}{\pi^2} \int \int d^2 \beta d^2 \tilde{\beta} K_{NA\tilde{N}\tilde{A}}(\beta, \tilde{\beta}; t) \left\langle \Psi(t) \right\rangle_\beta \otimes \left\langle \Psi(t) \right\rangle_{\tilde{\beta}} \]
Gravitational coupling strength in the granular case

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Let us study the dynamical evolution of the meta-system:

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\[ \left\langle \Psi(0) \right\rangle = \frac{1}{2} \left[ \left( \left| 0_{A} 1_{B} \right\rangle + \left| 1_{A} 0_{B} \right\rangle \right) \alpha \right\rangle \right] \otimes \left[ \left( \left| 0_{\tilde{A}} 1_{\tilde{B}} \right\rangle + \left| 1_{\tilde{A}} 0_{\tilde{B}} \right\rangle \right) \tilde{\alpha} \right\rangle \equiv \left| \psi(0) \right\rangle \otimes \left| \psi(0) \right\rangle \]

Schroedinger state at time \( t \)

\[ \left\langle \Psi(t) \right\rangle = \frac{1}{\pi^{2}} \int d^{2} \beta d^{2} \tilde{\beta} K^{N_{A}N_{\tilde{A}}}_{\beta, \tilde{\beta}} (\beta, \tilde{\beta}; t) \left\langle \psi(t) \right\rangle_{\beta} \otimes \left| \psi(t) \right\rangle_{\tilde{\beta}} \]

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Gravitational coupling strength in the granular case

\[ K_{G}^{gran} = \frac{2\hbar G \rho_{nucl}}{3\omega_{m}} \cong 10^{4} \times K_{G}^{hom} \]

Let us study the dynamical evolution of the meta-system:

Initial meta-state

\[ \left| \Psi(0) \right\rangle = \frac{1}{2} \left[ \left( 0_{A}1_{B} \right) + 1_{A}0_{B} \right) \alpha \right\rangle \otimes \left[ \left( 0_{\tilde{A}}1_{\tilde{B}} \right) + 1_{\tilde{A}}0_{\tilde{B}} \right) \tilde{\alpha} \right\rangle = \left| \psi(0) \right\rangle \otimes \left| \psi(0) \right\rangle \]

Schroedinger state at time \( t \)

\[ \left| \Psi(t) \right\rangle = \frac{1}{\pi} \int \int d^{2}\beta d^{2}\tilde{\beta} K_{N_{A}N_{\tilde{A}}}^{N_{A}N_{\tilde{A}}} (\beta, \tilde{\beta}; t) \left| \Psi(t) \right\rangle_{\beta} \otimes \left| \Psi(t) \right\rangle_{\tilde{\beta}} \]

Kernel

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\[ |\psi(t)\rangle_\beta = \frac{1}{\sqrt{2}} e^{-i\omega_{ph} t} \left( |0_A 1_B \rangle \otimes |\beta_c\rangle + f(\beta) |1_A 0_B \rangle \otimes |\beta_i\rangle \right) \]
\[
|\psi(t)\rangle_{\beta} = \frac{1}{\sqrt{2}} e^{-i\omega_{ph} t} \left( 0_A 1_B \right) \otimes |\beta_c\rangle + f(\beta) 1_A 0_B \otimes |\beta_l\rangle
\]

\[
f(\beta) = e^{i\kappa^2 (\omega_m^* t - \sin \omega_m^* t)} e^{i\kappa \text{Im} \left[ \beta \left( 1 - e^{-i\omega_m t} \right) \right]}
\]

\[
|\beta_c\rangle = |\beta e^{-i\omega_m^* t}\rangle
\]

\[
|\beta_l\rangle = |\beta e^{-i\omega_m^* t} + \kappa (1 - e^{-i\omega_m t})\rangle
\]
Interference visibility:

\[
|\psi(t)\rangle_{\beta} = \frac{1}{\sqrt{2}} e^{-i\omega_{ph}t} \left( |0_A1_B\rangle \otimes |\beta_c\rangle + f(\beta)|1_A0_B\rangle \otimes |\beta_l\rangle \right)
\]

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|\beta_c\rangle = |\beta e^{-i\omega_m^* t}\rangle
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\[
|\beta_l\rangle = |\beta e^{-i\omega_m^* t} + \kappa \left( 1 - e^{-i\omega_m^* t} \right)\rangle
\]

**INTERFERENCE VISIBILITY:**

\[
\text{Vis}(t) = 2 |\text{Tr}_{m,\tilde{m}} \text{Tr}_{\tilde{P}h} R^{(\alpha)}_V| \\
R^{(\alpha)}_V = \langle 1_A 0_B || \Psi \rangle \langle \Psi || 0_A 1_B \rangle
\]

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Visibility

\[ \frac{t \omega^*_m}{2\pi} \]
Interference visibility in the homogeneous case plotted for $\kappa = 1$, mass $= 5 \times 10^{-12}$ Kg, mirror size $L = 10^{-5}$ m and several values of the frequency: $\omega_m = \omega_m \exp \times 10^{-5}$ (dot-dashed), $\omega_m = \omega_m \exp \times 10^{-6}$ (dashed), $\omega_m = 5 \omega_m \exp \times 10^{-7}$ (dotted), $\omega_m = \omega_m \exp \times 10^{-7}$ (continuous black).
Interference visibility in the homogeneous case plotted for $\kappa=1$, mass=$5\times10^{-12}$ Kg, mirror size $L=10^{-5}$ m and several values of the frequency: $\omega_m = \omega_m \exp\times10^{-5}$ (dot-dashed), $\omega_m = \omega_m \exp\times10^{-6}$ (dashed), $\omega_m = 5\omega_m \exp\times10^{-7}$ (dotted), $\omega_m = \omega_m \exp\times10^{-7}$ (continuous black).
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We observe a more and more reduction of revival effect at the end of the cycle.
Comments:
1. if we include granularity effect we obtain that already for $\omega_m = \omega_m^{\text{exp}} \times 10^{-3}$ visibility behaves in a similar way to the homogeneous case with $\omega_m = \omega_m^{\text{exp}} \times 10^{-6}$.
2. in the free case of no NNG interaction, when $K_G = 0$ and $\alpha = 0$, we recover the result of Marshall et al., PRL 91 (2003) 130401:

$$\text{Vis}(t) = e^{i\kappa^2 (\omega_m t - \sin \omega_m t)} e^{-\kappa^2 (1 - \cos \omega_m t)}$$
Wigner function:
Calculation of Wigner function

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In order to monitor mirror’s state we compute the Wigner function. This quasi-distribution show both positive and negative parts, the latter being a signature of quantum coherence survival. So we expect that the action of NNG-induced decoherence, after some time, would reduce the interference patterns in the Wigner function.

*Wigner function:*
Calculation of Wigner function

In order to monitor mirror’s state we compute the Wigner function. This quasi-distribution show both positive and negative parts, the latter being a signature of quantum coherence survival. So we expect that the action of NNG-induced decoherence, after some time, would reduce the interference patterns in the Wigner function.

**Wigner function:**

\[
W(x, p; t) = \frac{1}{2\hbar \pi^2} \int d^2 \lambda e^{-\lambda \eta^* + \lambda^* \eta} \text{Tr} \left[ \rho_m(t) e^{\lambda \hat{b}^* - \lambda^* \hat{b}} \right]
\]

\[
\eta = \frac{ip}{\sqrt{2M\omega_m^* \hbar}} + x \sqrt{\frac{M\omega_m^*}{2\hbar}}
\]
\( \rho_m \) is the reduced density matrix of the mirror after a photon detection. We take this measurement as the process projecting the physical photon’s state onto the following state:
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|\varphi\rangle = \frac{1}{\sqrt{2}} \left( |0_A 1_B\rangle + |1_A 0_B\rangle \right)
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\[
\rho_m(t) = Tr_{Ph,\tilde{Ph},\tilde{m}} \left( |\varphi\rangle \langle \varphi | \otimes 1 \right) \Psi(t) \langle \Psi(t) | \right)
\]

|\varphi\rangle \equiv |\varphi\rangle \otimes |\tilde{\varphi}\rangle
\[ \rho_m \text{ is the reduced density matrix of the mirror after a photon detection. We take this measurement as the process projecting the physical photon's state onto the following state:} \]

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\[ |\varphi\rangle \equiv |\varphi\rangle \otimes |\tilde{\varphi}\rangle \]

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Wigner function
Wigner function
$W(x, p; t) = \frac{1}{4\hbar\pi^5} \int d^2 (\beta, \tilde{\beta}, \beta', \tilde{\beta}') e^{-2\eta\eta^*}$

$$\begin{bmatrix}
-\beta_c \beta_c^* - \frac{1}{2}|\beta_c|^2 - \frac{1}{2}|\beta_c'|^2 + 2\beta_c \eta^* + 2\beta_c^* \eta \\
\alpha_1 e \\
-\beta_i \beta_i'^* - \frac{1}{2}|\beta_i|^2 - \frac{1}{2}|\beta_i'|^2 + 2\beta_i \eta^* + 2\beta_i'^* \eta \\
+ \alpha_2 e \\
-\beta_c \beta_i'^* - \frac{1}{2}|\beta_c|^2 - \frac{1}{2}|\beta_i'|^2 + 2\beta_c \eta^* + 2\beta_i'^* \eta \\
+ \alpha_3 e \\
-\beta_i \beta_c'^* - \frac{1}{2}|\beta_i|^2 - \frac{1}{2}|\beta_c'|^2 + 2\beta_i \eta^* + 2\beta_c'^* \eta \\
+ \alpha_4 e
\end{bmatrix}$$

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Results for the homogeneous case, $\kappa=2$, $\omega_m = \omega_m \times 10^{-5}$, for the mirror size $L=10^{-5}$ m and for different intermediate times in a complete mirror oscillation.
Results for the homogeneous case, $\kappa=2$, $\omega_m = 5\omega_m \exp \times 10^{-7}$, for the mirror size $L=10^{-5}$ m and for different intermediate times in a complete mirror oscillation.
• in the first case, Wigner function after measurement is undistinguishable from the free case;
• in the second case, after a certain time a diminution of interference fringes is observed together with a contextual lowering of the first peak.
Prospects for future experimental tests
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Which technologies and devices are suitable for creating *macroscopically distinct* quantum state superpositions?
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Which technologies and devices are suitable for creating *macroscopically distinct* quantum state superpositions?

Optomechanical systems are very promising for making superpositions of even larger objects, such as microsized mirrors or cantilevers, and for testing quantum phenomena at larger scales (T. Kippenberg et al., Science 321 (2006) 1172; F. Marquardt et al., Physics 2 (2009) 40; I. Favero et al., Nature Phot. 3 (2009) 201; D. Kleckner et al., NJP 10 (2008) 095020).

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We computed the output of the mirror experiment proposed by Marshall et al. (PRL 91 (2009) 130401) within the framework of NNG model, assuming both homogeneous and granular mass distributions. By varying the experimental parameters in a wide range, a window of sensible parameters has been found in which the NNG induced decoherence effect is manifest.

Even if the experimental test of NNG has been proved to lie beyond current technology yet, we believe that due to the rapid progress in developing high-quality micro-optomechanical devices a prototypical experiment of this type could be soon realized.

We plan to include finite temperature effects in the NNG model; this is the first step towards a generalized model of gravity induced thermalization.

We plan to investigate quantum foundations of thermodynamics within the NNG model.

We will explore the possibility to set up possible experimental tests of NNG in the realm of ultracold atoms and superconductive devices.

F. Maimone, G. Scelza, A. Naddeo, V. Pelino, PRA 83 062124 (2011)